KINCOPPAL–ROSE BAY
SCHOOL OF THE SACRED HEART

**2010**
HIGHER SCHOOL CERTIFICATE
**HALF YEARLY EXAMINATION**

# Mathematics Extension 1

**General Instructions**

- Reading time – 5 minutes
- Working time – 2 hours
- Write using black or blue pen
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- All necessary working should be shown in every question
- Start a new booklet for each question

**Total marks – 72**

- Attempt Questions 1 – 6
- All questions are of equal value

**Question 1** (12 marks) Use a SEPARATE writing booklet **Marks**

(a) Solve for *x*, $\frac{3}{2-3x} \leq \frac{2}{3}$ **3**

(b) Solve the equation $\sin 2\theta = \cos\theta$ for $0 \leq \theta \leq 2\pi$ **3**

(c) Evaluate $\lim_{\theta \to 0} \frac{\tan\frac{\theta}{3}}{\theta}$ **2**

(d) If $f(x) = 2x^2 + x$, use the definition **2**

$$\frac{dy}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

to find the derivative of $f(x)$ at the point where $x = a$

(e) Find the coordinates of the point *P* which divides the interval *AB* externally in the ratio 4:1, if *A* is $(1,4)$ and *B* is $(3,-6)$ **2**

**End of Question 1**

**Question 2** (12 marks) Use a SEPARATE writing booklet **Marks**

(a) Use the substitution $u = 1 + x^2$ to evaluate $\int_1^{\sqrt{3}} 6x\sqrt{1+x^2}\,dx$ **3**

(b) Let $f(x) = \ln(\tan x)$, $0 < x < \frac{\pi}{2}$ **3**

Show that $f'(x) = 2\operatorname{cosec} 2x$

(c) (i) Express $\cos 3t - \sqrt{3}\sin 3t$ in the form $R\cos(3t+\alpha)$ for some $R > \alpha$ and $0 < \alpha < \frac{\pi}{2}$ **2**

(ii) Hence state the period and amplitude of $\cos 3t - \sqrt{3}\sin 3t$ . **2**

(d) The word EQUATIONS contains all five vowels. How many 7-letter 'words' consisting of all fives vowels can be formed from the letters of EQUATIONS? **2**

**End of Question 2**

**Question 3** (12 marks) Use a SEPARATE writing booklet **Marks**

(a) Prove by mathematical induction, for $n \geq 2$, that **3**

$$\left(1-\frac{1}{2^2}\right)\left(1-\frac{1}{3^2}\right)\left(1-\frac{1}{4^2}\right).....\left(1-\frac{1}{n^2}\right)=\frac{n+1}{2n}.$$

(b) Find the exact value of $\displaystyle\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \sin^2 3x \; dx$ **3**

(c) The polynomial $p(x)=x^3+ax^2+bx+12$ has a zero at $x=-1$ and has remainder 8 when divided by $x+2$. Find the constants $a$ and $b$. **3**

(d) If $\alpha, \beta$ and $\gamma$ are the roots of $x^3-6x^2-2x+4=0$, find the values of:

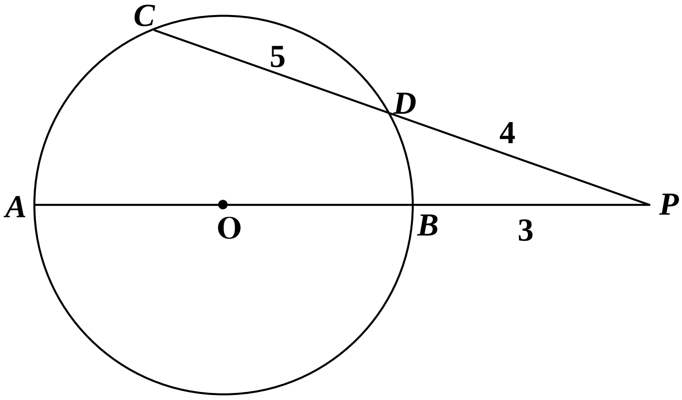
(i) $\alpha+\beta+\gamma$ **1**

(ii) $\alpha^2+\beta^2+\gamma^2$ **2**

**End of Question 3**

**Question 4** (12 marks) Use a SEPARATE writing booklet **Marks**

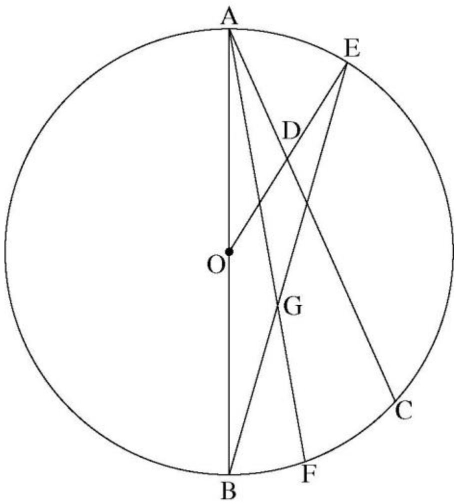
(a) **2**




$AB$ is the diameter of a circle, centre O.
$AB$ produced meets the secant $CD$ at $P$.
$CD = 5$, $DP = 4$ and $BP = 3$

Find the diameter of the circle.

(b) In the figure, *AOB* is the diameter of a circle centre *O*. *D* is a point on chord *AC* such that *DA* = *DO* and *OD* is produced to *E*. *AF* is the bisector of $\angle BAC$ and cuts *BE* in *G*.




Prove that:

(i) *GA* = *GB* **3**

(ii) *AOGE* is a cyclic quadrilateral **2**

(c) (i) Prove that $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$ **2**

(ii) Hence find $\int 2\sin^3\theta \ d\theta$ **3**



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**Question 5** (12 marks) Use a SEPARATE writing booklet **Marks**

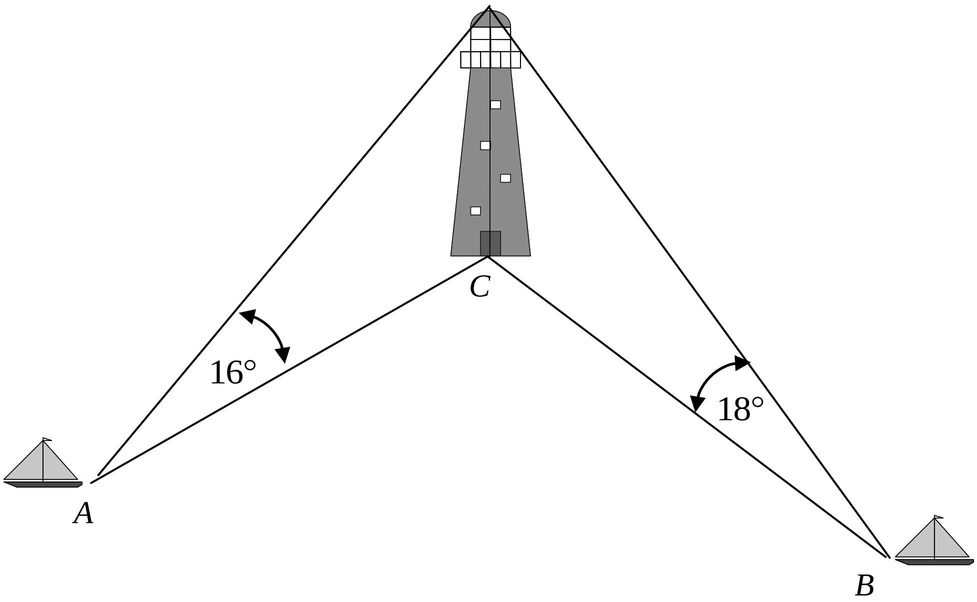
(a) A boat sails from a point *A* to a point *B*.

At point *A* the captain of the ship measures the angle of elevation of the top of a lighthouse as $16^\circ$ and the bearing of the lighthouse as $040^\circ$.

At point *B* the captain of the ship measures the angle of elevation of the top of the lighthouse as $18^\circ$ and the bearing of the lighthouse as $340^\circ$.

The top of the lighthouse is known to be 80 m above sea level.

The diagram below shows the angles of elevation of the top of the lighthouse from *A* and *B*.



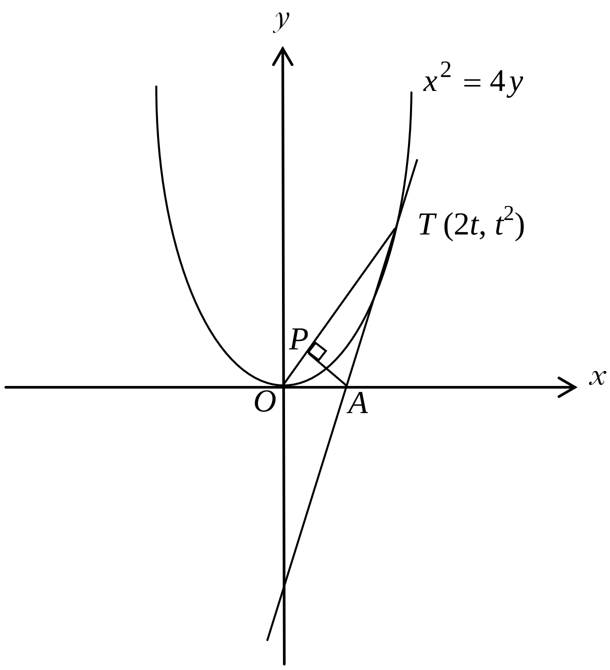

(i) Draw a bearing diagram showing the relative positions of *A*, *B* and *C* and use your diagram to explain why $\angle ACB = 60^\circ$. **1**

(ii) Hence, find the distance between *A* and *B*, correct to the nearest metre. **3**

(iii) Hence, find the bearing of *B* from *A*, to the nearest degree. **2**

**Question 5 continued on page 7**

**Question 5 continued** **Marks**




(b) The tangent at $T(2t, t^2)$, $t \neq 0$, on the parabola $x^2 = 4y$ meets the $x$ axis at $A$.

$P(x, y)$ is the foot of the perpendicular from $A$ to $OT$, where $O$ is the origin.

The equation of the tangent at $T$ is $y = tx - t^2$

(i) Prove that the equation of $AP$ is $y = -\frac{2}{t}(x - t)$ **2**

(ii) Show that the equation of $OT$ is $t = \frac{2y}{x}$ **1**

(iii) Hence, or otherwise, prove that the locus of $P(x, y)$ lies on a circle with centre $(0,1)$ and give its radius. **3**

**End of Question 5**

**Question 6** (12 marks) Use a SEPARATE writing booklet **Marks**

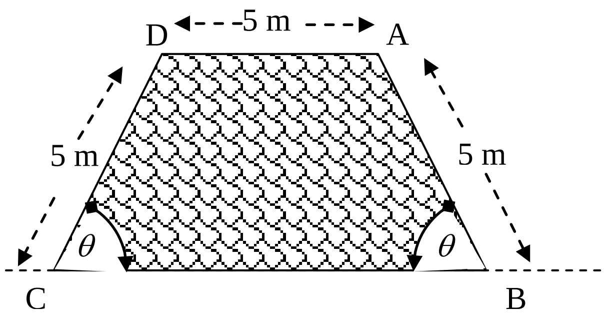
(a) (i) Draw a neat sketch of the function, $y = \log_e(x-1)$. **1**

(ii) This function meets the line $y = 2$ at the point $P$ and the $x$- axis at the point $Q$. Show that the coordinates of P and Q are $(e^2+1,\ 2)$ and $(2,0)$ respectively. **2**

(iii) If $S$ is the point (0,2), find the co-ordinates of the point $R$ if $OSPR$ is a rectangle. Label the points $S$ and $R$ on your sketch. **1**

(iv) Show that the arc $PQ$, divides the rectangle $OSPR$ into two regions of equal area. **3**

(b) The illustration below is part of the cross section of the roof of the Mathematics Faculty staffroom.




(i) If $\angle ABC = \angle DCB = \theta$ show that the area of this cross section is given by: **2**

$$A = 25\sin\theta(1+\cos\theta)$$

(ii) Given $\dfrac{dA}{d\theta} = 50\cos^2\theta + 25\cos\theta - 25$ **3**

and $\dfrac{d^2A}{d\theta^2} = -100\cos\theta\sin\theta - 25\sin\theta$

Find the value of $\theta$ which will make this area a maximum.

**End of Examination**



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SOLUTIONS

# Mathematics Extension 1

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| Question | Answer | Marks |
| --- | --- | --- |
| 1(a) | $\dfrac{3}{2-3x} \le \dfrac{2}{3}$<br><br>$2-3x \neq 0$<br>$\therefore x = \dfrac{2}{3}$ ✓<br><br>$\dfrac{3}{2-3x} = \dfrac{2}{3}$<br>$9 = 4-6x$<br>$-6x = 5$<br>$x = \dfrac{-5}{6}$ ✓<br><br>Test points ✓ $\dfrac{-5}{6}$ ☒ $\dfrac{2}{3}$ ✓<br><br>$\therefore \quad x \le \dfrac{-5}{6} \quad and \quad x > \dfrac{2}{3}$ ✓ | 3 |
| 1(b) | $\sin 2\theta = \cos\theta$<br>$2\sin\theta\cos\theta = \cos\theta$<br>$2\sin\theta\cos\theta - \cos\theta = 0$<br>$\cos\theta(2\sin\theta - 1) = 0$ ✓<br><br>$\cos\theta = 0$ $\quad or \quad$ $2\sin\theta - 1 = 0$<br>$\theta = \dfrac{\pi}{2} \; or \; \dfrac{3\pi}{2}$ ✓ $\quad$ $\sin\theta = \dfrac{1}{2}$<br>$\theta = \dfrac{\pi}{6} \; or \; \dfrac{5\pi}{6}$ ✓ | **3** |
| 1(c) | $\lim\limits_{\theta\to 0} \dfrac{\tan\frac{\theta}{3}}{\theta} = \dfrac{\tan\frac{\theta}{3}}{\frac{\theta}{3}} \times \dfrac{\frac{\theta}{3}}{\theta}$ ✓<br>$= 1 \times \dfrac{1}{3}$<br>$= \dfrac{1}{3}$ ✓ | **2** |

| | | |
|---|---|---|
| 1(d) | $\frac{dy}{dx} = \frac{f(x+h)-f(x)}{h}$<br>$= \frac{2(x+h)^2+(x+h)-\left(2x^2+x\right)}{h}$<br>$= \frac{4xh+h^2+h}{h}$<br>$= \frac{h(4x^2+h+1)}{h}$ ✓<br>$\lim_{x\to 0} = 4x^2+0+1$<br>$= 4x^2+1$ ✓ | **2** |
| 1(e) | $(1,4)$ *and* $(3,-6)$ *externally* $4:1$<br>$\therefore \frac{1\times 1+-4\times 3}{1-4} \ , \ \frac{1\times 4+-4\times -6}{1-4}$ ✓<br>$\therefore P\left(\frac{11}{3}, \ \frac{-28}{3}\right)$ ✓ | **2** |

| Question | Answer | Marks |
|---|---|---|
| 2(a) | $= \int_{1}^{\sqrt{3}} 6x\sqrt{1+x^2}\ dx \qquad u = 1+x^2$<br>$= \int 6x\sqrt{u}\ \frac{du}{2x} \qquad \frac{du}{dx} = 2x$<br>$= \int 3\sqrt{u}\ du \qquad \therefore dx = \frac{du}{2x}$<br>$= \int 3u^{\frac{1}{2}}\ du$ ✓<br>$= \left[\frac{3u^{\frac{3}{2}}}{\frac{3}{2}}\right]$<br>$= \left[2(1+x^2)^{\frac{3}{2}}\right]_1^{\sqrt{3}}$ ✓<br>$= \left[2\left(1+(\sqrt{3})^2\right)^{\frac{3}{2}}\right] - \left[2(1+1^2)^{\frac{3}{2}}\right]$<br>$= 16 - 4\sqrt{2}$ ✓ | 3 |
| 2(b) | $f(x) = \ln(\tan x)$<br>$f'(x) = \frac{\sec^2 x}{\tan x}$ ✓<br>$= \frac{1}{\cos^2 x} \times \frac{\cos x}{\sin x}$<br>$= \frac{1}{\cos x \sin x}$<br>$= \frac{1}{\frac{1}{2}\sin 2x}$ ✓<br>$= 2\cos ec 2x$ ✓ | **3** |
| 2(c)(i) | $\cos 3t - \sqrt{3}\sin 3t$<br>$\therefore R = \sqrt{1^2 + \sqrt{3}^2} = 2$<br>$\tan\alpha = \frac{\sqrt{3}}{1} \quad hence \quad \alpha = \frac{\pi}{3}$ ✓<br>$\therefore \cos 3t - \sqrt{3}\sin 3t = 2\cos\left(3t + \frac{\pi}{3}\right)$ ✓ | **2** |

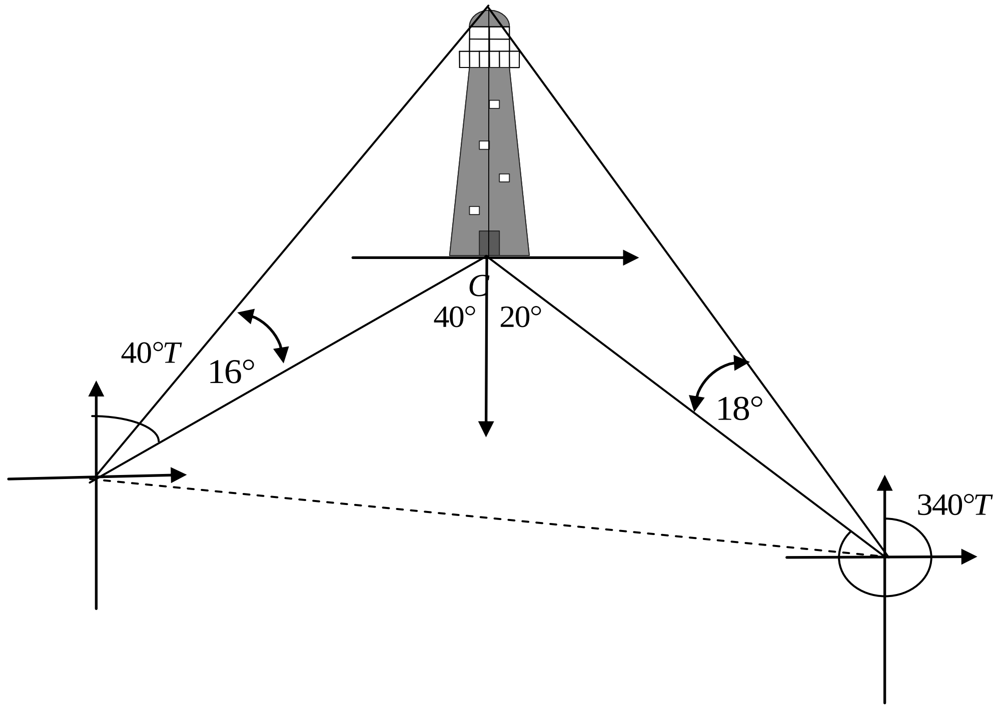
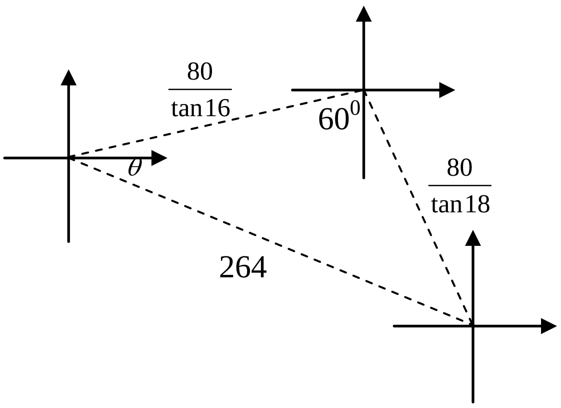
| | | |
|---|---|---|
| 2(c)(ii) | $\cos 3t - \sqrt{3}\sin 3t = 2\cos\left(3t + \frac{\pi}{3}\right)$<br><br>$\therefore amplitude = 2$ ☑<br>$period = \frac{2\pi}{3}$ ☑ | **2** |
| 2(d) | $Equations = 9\ letters\ \ (5\ vowels,\ 4\ consonants)$<br><br>$\therefore 7\ letters = 5\,vowels\ and\ 2\ consonants$<br>$\therefore\ {}^{5}P_{5} \times {}^{4}P_{2}$ ☑<br>$= 1440\ \ ways$ ☑ | **2** |

| Question | Answer | Marks |
|---|---|---|
| 3(a) | $\left(1-\frac{1}{2^2}\right)\left(1-\frac{1}{3^2}\right)\left(1-\frac{1}{4^2}\right).....\left(1-\frac{1}{n^2}\right)=\frac{n+1}{2n}$<br><br>$step 1:\ prove\ true\ for\ n=2$<br>$LHS=1-\frac{1}{2^2}=\frac{3}{4}\qquad RHS=\frac{2+1}{2\times 2}=\frac{3}{4}$<br>$\therefore LHS=RHS\ \ (hence\ true\ for\ n=2)$ ✓<br><br>$Step\ 2:\ assume\ true\ for\ n=k$<br>$\left(1-\frac{1}{2^2}\right)\left(1-\frac{1}{3^2}\right)\left(1-\frac{1}{4^2}\right).....\left(1-\frac{1}{k^2}\right)=\frac{k+1}{2k}$<br><br>$Step\ 3:\ prove\ true\ for\ n=k+1$<br>$\boxed{RTP:\left(1-\frac{1}{2^2}\right)\left(1-\frac{1}{3^2}\right)\left(1-\frac{1}{4^2}\right).....\left(1-\frac{1}{k^2}\right)\left(1-\frac{1}{(k+1)^2}\right)=\frac{k+2}{2(k+1)}}$<br>$LHS=\left(1-\frac{1}{2^2}\right)\left(1-\frac{1}{3^2}\right)\left(1-\frac{1}{4^2}\right).....\left(1-\frac{1}{k^2}\right)\left(1-\frac{1}{(k+1)^2}\right)$<br>$=\frac{k+1}{2k}\left(1-\frac{1}{(k+1)^2}\right)$<br>$=\frac{k+1}{2k}\left(\frac{(k+1)^2-1}{(k+1)^2}\right)$<br>$=\frac{k+1}{2k}\left(\frac{k(k+2)}{(k+1)^2}\right)$<br>$=\frac{1}{2}\left(\frac{k+2}{(k+1)}\right)$<br>$=\frac{k+2}{2(k+1)}$<br>$=RHS$<br>$\therefore true\ \ for\ \ n=k+1$ ✓<br><br>$Step\ 4:$ since true for n=2 and n=2+1=3, ... and true for n = k and n = k+1<br>therefore it is true for $n\geq 2$ ✓ | 3 |

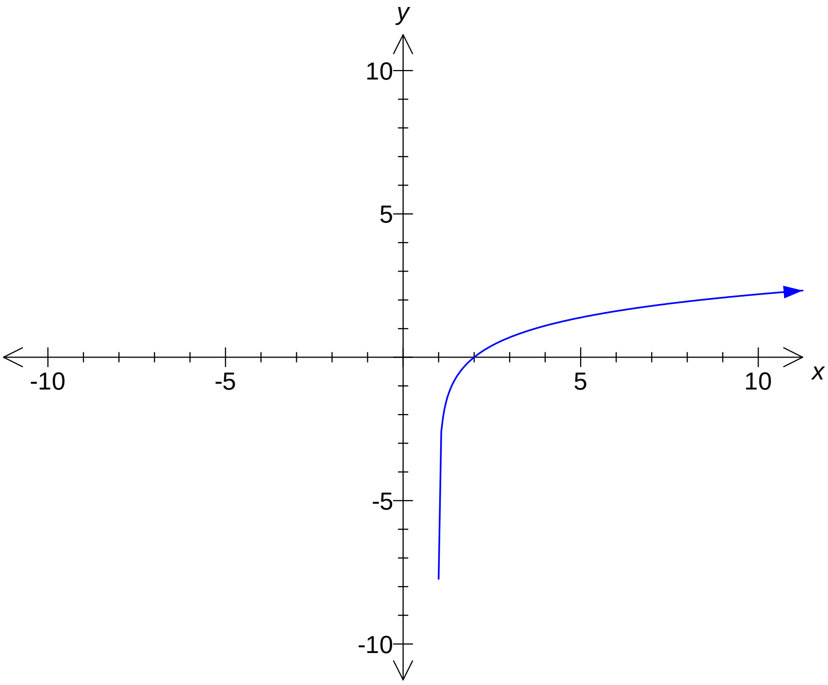
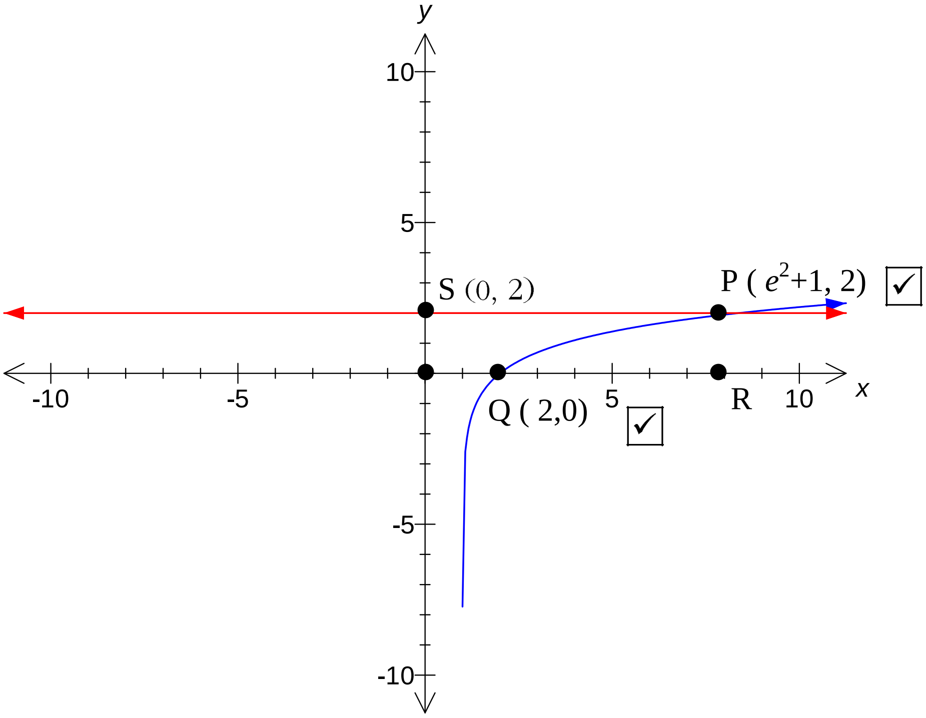
| | | |
|---|---|---|
| 3(b) | $\int \sin^2 3x\ dx = \frac{1}{2}x - \frac{1}{12}\sin 6x$ ☑<br>$\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \sin^2 3x\ dx = \left[\frac{1}{2}x - \frac{1}{12}\sin 6x\right]_{\frac{\pi}{4}}^{\frac{\pi}{3}}$<br>$= \left[\frac{1}{2}\left(\frac{\pi}{3}\right) - \frac{1}{12}\sin 6\left(\frac{\pi}{3}\right)\right] - \left[\frac{1}{2}\left(\frac{\pi}{4}\right) - \frac{1}{12}\sin 6\left(\frac{\pi}{4}\right)\right]$ ☑<br>$= \left(\frac{\pi}{6} - 0 - \frac{\pi}{8} - \frac{1}{12}\right)$<br>$= \frac{\pi}{24} - \frac{1}{12}$ ☑ | **3** |
| 3(c) | $p(x) = x^3 + ax^2 + bx + 12$<br>$p(-1) = -1 + a - b + 12 = 0 \quad \therefore a - b = -11 \quad (i)$<br>$p(-2) = -8 + 4a - 2b + 12 = 8 \quad \therefore 4a - 2b = 4 \quad (ii)$ ☑<br>$(ii) - 2(i): \quad 2a = 26$<br>$\boxed{a = 13}$ ☑<br>$sub\ a = 13\ \text{int}\,o\ (i)$<br>$\therefore\ 13 - b = -11$<br>$-b = -24$<br>$\boxed{b = 24}$ ☑ | **3** |
| 3(d) | $x^3 - 6x^2 - 2x + 4 = 0$<br>$(i) \quad \alpha + \beta + \gamma = \frac{-b}{a} = 6$ ☑<br>$(ii) \quad \alpha^2 + \beta^2 + \gamma^2$<br>$= (\alpha + \beta + \gamma)^2 - 2(\alpha\beta + \beta\gamma + \alpha\gamma)$ ☑<br>$= (6)^2 - 2(-2)$<br>$= 36 + 4$<br>$= 40$ ☑ | **3** |

| Question | Answer | Marks |
| --- | --- | --- |
| 4(a) | Using similar triangles:<br>$\frac{4}{d+3} = \frac{3}{9}$ $(where\ d = diameter)$ ☑<br>$3d + 9 = 36$<br>$3d = 27$<br>$d = 9$<br>$\therefore diameter = 9$ ☑ | **3** |
| 4(b)(i) | $OB = OE$ $(equal\ radii)$<br>$\angle OBE = \angle OEB$ $(base\ \angle 's\ of\ isoceles\ \Delta\ are\ =)$ ☑<br>$\angle OEB = \angle BAC$ $(angles\ s\tan ding\ on\ same\ arc\ are\ equal)$ ☑<br><br>$\therefore \angle OBE = \angle BAC$<br>$\therefore \Delta AGB\ is\ isoceles (base\ \angle 's\ of\ isoceles\ \Delta\ are\ =)$ ☑<br>$\therefore AG = GB$ | **3** |
| 4(b)(ii) | $\angle OAG = \angle GEO$ $(angles$ standing $on\ same\ arc\ are\ equal)$<br>$label\ X$ (intersection $of\ AF\ and\ OE)$<br><br>$\therefore \angle AXO = \angle EXG$ $(vertically\ opp\ \angle 's\ =)$ ☑<br>$\therefore \angle AOX = \angle EGX$ (angle sum of a $\Delta$ is supplementary)<br><br>$\sin ce$ $\angle AOX = \angle EGX$<br>$\therefore$ angles standing on the same arc are equal ☑<br>$\therefore AOGE$ is a cyclic quadrilateral | **2** |
| 4(c)(i) | $\sin 3\theta = \sin(2\theta + \theta)$<br>$= \sin 2\theta \cos\theta + \sin\theta \cos 2\theta$ ☑<br>$= (2\sin\theta\cos\theta)(\cos\theta) + \sin\theta(1 - 2\sin^2\theta)$<br>$= 2\sin\theta\cos^2\theta + \sin\theta - 2\sin^3\theta$<br>$= 2\sin\theta(1-\sin^2\theta) + \sin\theta - 2\sin^3\theta$ ☑<br>$= 2\sin\theta - 2\sin^3\theta + \sin\theta - 2\sin^3\theta$<br>$= 3\sin\theta - 4\sin^3\theta$ | **2** |

| | | |
|---|---|---|
| 4(c)(ii) | $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$<br>$4\sin^3\theta = 3\sin\theta - \sin 3\theta$<br>$\sin^3\theta = \frac{1}{4}(3\sin\theta - \sin 3\theta)$ ☑<br><br>$\int 2\sin^3\theta \; d\theta = 2\int \sin^3\theta \; d\theta$<br>$= 2\int \frac{1}{4}(3\sin\theta - \sin 3\theta)$ ☑<br>$= \frac{1}{2}\int (3\sin\theta - \sin 3\theta)$<br>$= \frac{1}{2}\left[-3\cos\theta + \frac{1}{3}\cos 3\theta\right]$<br>$= \frac{-3}{2}\cos\theta + \frac{1}{6}\cos 3\theta + C$ ☑ | **3** |

| Question | Answer | Marks |
|---|---|---|
| 5(a)(i) | <br>☑ *alternate* $\angle$*'s =*<br>$\therefore\ \angle ACB = 20 + 40 = 60°$ | **1** |
| 5(a)(ii) | *Let* $AC = x$ , $BC = y$ *and* $CT = 80\ m$<br>$\therefore \tan 16 = \dfrac{80}{x}$ , $\tan 18 = \dfrac{80}{y}$ *and* $AB^2 = x^2 + y^2 - 2xy\cos 60$ ☑<br>$AB^2 = \left(\dfrac{80}{\tan 16}\right)^2 + \left(\dfrac{80}{\tan 18}\right)^2 - 2\left(\dfrac{80}{\tan 16}\right)\left(\dfrac{80}{\tan 18}\right)\cos 60$<br>$AB^2 = 80^2\left(\dfrac{1}{\tan^2 16} + \dfrac{1}{\tan^2 18} - \dfrac{2\cos 60}{\tan 16 \tan 18}\right)$<br>$AB^2 = 69766.6396$ ☑<br>$AB = \sqrt{69766.6396}$<br>$AB = 264\ m$ (*nearest metre*) ☑ | **3** |
| 5(a)(iii) | <br>$\dfrac{\sin\theta}{\left(\frac{80}{\tan 18}\right)} = \dfrac{\sin 60}{264}$<br>$\therefore \theta = \sin^{-1}\left(\dfrac{\sin 60}{264} \times \dfrac{80}{\tan 18}\right)$<br>$\theta = 54°$ (*nearest degree*) ☑<br>$\therefore$ *Bearing of B from A*<br>$= 54 + 40 = 090°T$ ☑ | **2** |

| | | |
|---|---|---|
| 5(b)(i) | $m_{OT} = \frac{t^2-0}{2t-0} = \frac{t}{2}$<br>$AP \perp OT$<br>$\therefore m_{AP} = -\frac{2}{t}$ ☑<br>$A$ *is where* tangent $y = tx - t^2$ *crosses* $x$ *axis* $\therefore A\ (t,0)$ ☑<br>$\therefore$ *equation* $AP:\ y - 0 = -\frac{2}{t}(x-t)$<br>$y = -\frac{2}{t}(x-t)$ | **2** |
| 5(b)(ii) | $m_{OT} = \frac{t^2-0}{2t-0} = \frac{t}{2}$<br>$\therefore$ *Equation* $OT$ *is*: $y - t^2 = \frac{t}{2}(x-2t)$ ☑<br>$y - t^2 = \frac{tx}{2} - t^2$<br>$y = \frac{tx}{2}$<br>$\therefore$ $tx = 2y$<br>$t = \frac{2y}{x}$ | **1** |
| 5(b)(iii) | $P$ *lies on the line* $AP: y = -\frac{2}{t}(x-t)$<br>*and* $t = \frac{2y}{x}$<br>$\therefore\ y = \frac{-2}{\frac{2y}{x}}\left(x - \frac{2y}{x}\right)$ ☑<br>$y = \frac{-2x}{2y}\left(x - \frac{2y}{x}\right)$<br>$y = \frac{-x}{y}\left(x - \frac{2y}{x}\right)$<br>$y^2 = -x^2 + 2y$<br>$x^2 - 2y + y^2 = 0$<br>$(x-1)^2 + y^2 = 1$<br>$\therefore$ *circle with centre* $(1,0)$ ☑<br>$\therefore$ *radius* $= 1$ ☑ | **3** |

| Question | Answer | Marks |
|---|---|---|
| 6(a)(i) |  | **1** |
| 6(a)(ii) |   | **2** |
| 6(a)(iii) | $R\ (e^2+1,\ 0)$ ✓ | **1** |

| | | |
|---|---|---|
| 6(a)(iv) | $Area\ rectangle = L \times B = 2(e^2+1)$<br><br>$y = \log_e(x-1) \quad \therefore x = e^y + 1$<br>$\int_0^2 e^y + 1\ dy = \left[e^y + y\right]_0^2$ ✓<br>$= \left[(e^2+2)-(e^0+0)\right]$<br>$= \left[(e^2+2)-(e^0+0)\right]$<br>$= \left[(e^2+2)-1\right]$<br>$= e^2+1$ ✓<br><br>$\therefore\ Area\ arc\ PQ = e^2+1$<br>$Area\ arc = \frac{1}{2}(area\ rectangle)$ ✓ | **3** |
| 6(b)(i) | Area of trapezium = area rectangle + 2 area of triangles<br><br>$A = 5y + 2\times\frac{1}{2}xy \quad where\ \sin\theta = \frac{y}{5} \Rightarrow y = 5\sin\theta$ ✓<br>$\cos\theta = \frac{x}{5} \Rightarrow x = 5\cos\theta$<br>$\therefore A = 25\sin\theta + (5\cos\theta)(5\sin\theta)$<br>$A = 25\sin\theta(1+\cos\theta)$ ✓ | **2** |
| 6(b)(ii) | $Max\ occurs\ when \frac{dA}{d\theta} = 0$<br>$\therefore \frac{dA}{d\theta} = 50\cos^2\theta + 25\cos\theta - 25 = 0$<br>$25(2\cos^2\theta + \cos\theta - 1) = 0$<br>$25(2\cos\theta - 1)(\cos\theta + 1) = 0$<br>$\therefore 2\cos\theta - 1 = 0 \quad or \quad \cos\theta + 1 = 0$<br>$2\cos\theta = 1 \qquad \cos\theta = -1$<br>$\theta = 60\ or\ 300 \qquad \theta = 270$ ✓<br>since θ is acute $\therefore test\ \ \theta = \frac{\pi}{3}$ ✓<br>$\frac{d^2A}{d\theta^2} = -100\cos 60\sin 60 - 25\sin 60 < 0 \qquad \therefore\ concave\ up$ since $\frac{d^2A}{d\theta^2} < 0$<br>$\therefore$ max of $at\ \theta = \frac{\pi}{3}$ ✓ | **3** |